Algoritmo Modificado

$\sqrt{3351}$	50	
2500	$\times$ 2	
851	$100 \times 8 = 800$	$100 \times 7 = 700$
800	Teste : $8^2 = 64$	Teste : $7^2 = 49$
51	(não serve)	Raiz $= 50 + 7 = 57$
100+		
151		
49−		
R = 102		

Prova da identificação dos algoritmos

Da exposição resulta que para a extração da raiz quadrada de um número N, subtrae-se de N o quadrado de Z, procura-se um número q tal que seja o quociente de N pelo *dôbro de* Z, e ainda faz-se o teste da nova sucessão de ímpares, cuja soma é q^2, isto é, q é tal que $2Z \cdot q + q^2$ é o maior número inferior ao resto $N - Z^2$, que é justamente o que se faz no algoritmo tradicional: $(2Z + q)\,q$.

O que se fêz foi simplesmente separar o cálculo q^2 do teste para uma melhor aprendizagem.

Exemplo:

$\sqrt{3331}$	50		
2500	$\times$ 2		
851	100	100	Raiz $= 50 + 7 = 57$
749	8+	7+	
102	108	107	
	$\times$ 8	$\times$ 7	
	864	749	

Ordered semigroups which contain zeroid elements

by **C. W. Leininger**

In [1] Clifford and Miller show that if a semigroup S has a zeroid element, then then its kernel is the subgroup K of zeroids of S. Furthermore K determines a partition G of S in a certain way. The purpose of this paper is to consider such a semigroup under the suppositions that K is a nondegenerate subset of S and there is a comparable pair of elements of S not both in the same set of G. We find that K inclndes a subchain Q of S which is o-isomorphic to the additive group of integers. Some aspects of the structure of ordered semigroups with zeroids elements are then investigated.

1. Introduction.

If z denotes the identity element of K, then the subsemigroug J such that

$$J = \{x : x \in S,\ xz = zx = z\}$$

is called the core of S and the set $J \cup K$ is called the frame of S. For convenience we summarize from [1, p. 121] the following properties pertaining to the gross structure of S:

P1. If $J(u) = \{x : x \in S,\ u \in K,\ xz = zx = u\}$, then the collection of the sets $J(u)$ is the partition G (note $J(z) = J$).

P2. If $a \in J(u)$ and $b \in J(v)$, then $ab \in J(uv)$.

P3. If $a \in J(u)$ and $v \in K$, then $av = uv$ and $va = vu$.

If S is ordered, it is apparent that it also possesses the following properties:

P4. If $a \in J(u)$, $b \in J(v)$ and $u < v$, then $b \not\leq a$.

P5. If $a \in J(u)$, $b \in J(v)$, $u \neq v$ and $a < b$, then $u < v$.

P6. If a and b are in $J(u)$ and $a < c < b$, then $c \in J(u)$.

2. Semigroups with the comparability condition.

It is readily established that no finite semigroup with zeroid elements has this property.

THEOREM 1. *Suppose* S *is an ordered semigroup containing zeroids* u *and* v, *and there is a pair* a, b *of elements of* S *such that* a *is in* J(u), b *is in* J(v) *and* a < b. *Then there is an infinite, cyclic and totally ordered subgroup* Q *of the kernel* K *of* S *and a subsemigroup* T *of* S *such that* Q *is the kernel of* T.

PROOF. As the conditions of P5 are met, we have $u < v$. Since $u^{-1} \neq v^{-1}, z = uu^{-1} < vu^{-1}$. Let q denote vu^{-1}. Then $z < q$ implies $q < q^2$ since q is not idempotent. It follows that for each positive integer $n, q^n \in K$ and $q^n < q^{n+1}$, and since $q^{-1} < z$, it also follows that $q^{-n} \in K$ and $q^{-(n+1)} < q^{-n}$. If Q denotes the chain generated by q and $T = \{x : x \in J(v), v \in Q\}$, then by [1, pp. 121-123], T is a semigroup with Q its kernel.

An example of a totally ordered semigroug which is not a group is given by $(S, +, <)$, where $S = I \cup x$, I is the additive group of integers with the natural ordering, $x + x = x, x + n = n + x = n$ for each integer n, and $0 < x < 1$.

3. Structure theorems.

It follows from P3 that the frame of S is a subsemigroups of S. Hence if S satisfies the conditions of Theorem 1 and J is a subchain of S, it follows from P6 that $J \cup Q$ is also a suchain of S. It is known [1, p. 120, Theorem 3] that the mapping $a \to za(za)$, $a \in S$, is a homomorphism μ of S onto K. We apply the latter concept to an ordered semigroup.

THEOREM 2. *If* S *is an ordered semigroup containing zeroid elements, and if its kernel* K *is a subchain* Q *of* S *which is generated by the zeroid* q, *then* μ *is the only* o-*homomorphism of* S *onto* K.

PROOF. Denote by λ an o-homomorphism of S onto K. Since $\lambda(z) = z$, if $u \in Q$ and $a \in J(u)$, then $\lambda(u) = \lambda(za) = z\lambda(a) = \lambda(a)$. Furthermore, since Q is o-isomorphic to the additive group I of integers and the identity mapping is the only o-automorphism of I, it follows that $\lambda(a) = za$.

We note that in the above example S is not Archimedean and contains no anomalous pair as defined by [2, p. 162].

THEOREM 3. *Suppose* S *is a totally ordered Archimedean semigroup containing zeroid elements. Then* (i) J (z) = z *and* (ii) if S≠K, *then* S *contains an anamalous pair.*

PROOF. (i) If $x \in J(x), x \neq z$, then by P2, $x^n \in J(z^n) = J(z), n = 1, 2, 3 \cdots$. According to P6 and Theorem 1 no interval of $J(z)$ contains the generator q of the subgroup Q of K. But there is an interval of $J(z)$ which contains z and x^n. Hence $x^n < q$.

(ii) Since $J(z) = z$, P4 implies that a, b is an anomalous pair if there is a u in K such that $a \in J(u)$ and $b \in J(u)$.

It may be observed that if $K = Q$, as in the example, then Theorem 3 (i) takes the following stronger form.

THEOREM 4· *Suppose* S *is a totally ordered semigroup containing zeroid elements and the kernel* K *of* S *is cyclic. Then* S *is Archimedean if and only if* J (z) = z.

PROOF. If S is Archimedean, then $J(z) = z$ according to Theorem 3. Suppose $J(z) = z$. It follows from Theorem 1 that K is an infinite subchain of S. Hence if $u \in K$, $u > z$ and m is a positive integer, there is a positive integer n such that $u^n > u^m$. Since if $a \in J(u)$, then $a^n \in J(u^n)$, it follows from P4 that $a^n > u^m$. If $u < z$, it can be shown similarly that $a^{-n} < u^{-m}$.

REFERENCES

[1] A. H. CLIFFORD and D. D. MILLER, *Semigroups having zeroid elements*, Amer. J. Math. **70** (1948), pp. 117-125.

[2] L. FUCHS, *Partially ordered algebraic systems*, Addison-Wesley, Reading, Mass., U. S. A., 1963.

Matrices whose sum is the identity matrix

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1. Let $A_i (i = 1, \cdots, m)$ be $n \times n$ complex matrices and let n_i denote the rank of A_i. In [1], p. 68 the following problem is posed:

If the matrices $A_i (i = 1, \cdots, m)$ *are symmetric and* $\sum_{i=1}^{m} A_i = E$ *(*E *denotes the* $n \times n$ *identity matrix), show that the following statements are equivalent:*

a) $A_i^2 = A_i \qquad (i = 1, \cdots, m)$

b) $\sum_{i=1}^{m} n_i = n$

c) $A_i A_j = 0 \qquad (i, j = 1, \cdots, m;\ i \neq j)$.

In [3] it is asked whether it is possible to drop the condition that the matrices $A_i (i = 1, \cdots, m)$ should be symmetric. In the present note we answer this question in the affirmative. So we no longer assume that the matrices $A_i (i = 1, \cdots, m)$ are symmetric.

We prove first that a) implies b).

If A_i is idempotent there exists a nonsingular matrix T_i such that (see [2], Vol. I, p. 226)

$$(1) \qquad A_i = T_i \operatorname{diag}(\overbrace{1, \cdots, 1}^{n_i}, \overbrace{0, \cdots, 0}^{n-n_i}) T_i^{-1}.$$

This can be proved very easily if we note that A_i is idempotent if and only if each diagonal block in its JORDAN normal form