

Geodesic curvature of a curve of a vector field

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1. Introduction.

Let V be a vector field in a surface in a Euclidean space of three dimensions. PAN [3] has studied the normal curvature of the vector field V with generalization obtained by the author [2].

The object of the present note is to define the geodesic curvature of the curve of the vector field and obtain some properties. As a special case when the curves of the vector field V form an orthogonal net of co-ordinate curves, these geodesic curvatures have the known form.

2. Consider upon a surface S

$$x^i = x^i(u^1, u^2) \quad (i = 1, 2, 3),$$

a curve C defined by

$$u^\alpha = u^\alpha(s) \quad (\alpha = 1, 2).$$

With each point of the surface we associate an arbitrary but fixed vector field V . The components v^i and p^α of the vector field V in the x 's and u 's are connected by the relation

$$v^i = x^i_{,\alpha} p^\alpha.$$

A curve on the surface S along which the vectors of the vector field V are tangent is

called the curve of the vector field. It is defined by [3]

$$\varepsilon_{\alpha\beta} p^\alpha du^\beta = 0.$$

The geodesic curvature of the curve C_V of the vector field V shall, therefore, be defined by [1]

$$(2.1) \quad v^k g = -\frac{1}{\sqrt{g}} \frac{\partial}{\partial u^\beta} \left(\frac{\sqrt{g} g^{\alpha\beta} \varepsilon_{\lambda\alpha} p^\lambda}{(g^{\gamma\delta} \varepsilon_{\alpha\gamma} \varepsilon_{\mu\delta} p^\alpha p^\mu)^{1/2}} \right)$$

Use of formulae

$$(2.2) \quad g^{\alpha\beta} \varepsilon_{\alpha\gamma} \varepsilon_{\beta\delta} = g_{\gamma\delta}$$

$$(2.3) \quad \varepsilon_{\lambda\alpha} g^{\beta\alpha} = \varepsilon^{\gamma\beta} g_{\gamma\lambda}$$

in (2.1) yields the relation

$$v^k g = -\frac{1}{\sqrt{g}} \frac{\partial}{\partial u^\beta} \left(\frac{e^{\gamma\beta} g_{\gamma\lambda} p^\lambda}{(g_{\alpha\mu} p^\alpha p^\mu)^{1/2}} \right).$$

In particular when p^α are the components of a unit vector, we have

$$(2.4) \quad -v^k g = \varepsilon^{\alpha\beta} p_{\alpha;\beta}$$

where semi-colon (;) followed by an index denotes covariant differentiation with respect to u with that index. Since the right hand expression of (2.4) is a scalar called the curl of the vector p_α [3], we have

