Olinde Rodrigues, mathematician and social reformer*

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Olinde Rodrigues lived in one of the most interesting periods for the development of mathematics in France. The foundation of the *École Polytechnique* in 1794, one year before Rodrigues's birth, had changed at a stroke the study of the subject. It was the great geometer Gaspard Monge, a friend of Napoleon, who founded the school and created its ethos as a centre of engineering studies based on a thorough mathematical foundation. Its pupils were selected entirely on the basis of their mathematical ability: later luminaries like Cauchy and Arago were in the first cohort. Equally important was the opening of free secondary education to all. Still on mathematics, an old pupil of the *Polytechnique*, Joseph-Diaz Gergonne, founded in 1810 the first journal purely devoted to mathematics, the *Annales de Mathématiques Pures et Appliquées* which, although soon discontinued, was revived in 1836 by Joseph Liouville as the *Journal de Mathématiques Pures et Appliquées*, popularly called *Journal de Liouville*, just as the mathematical journal that August Leopold Crelle had founded in Germany in 1826 was referred to as the *Crelle Journal*.

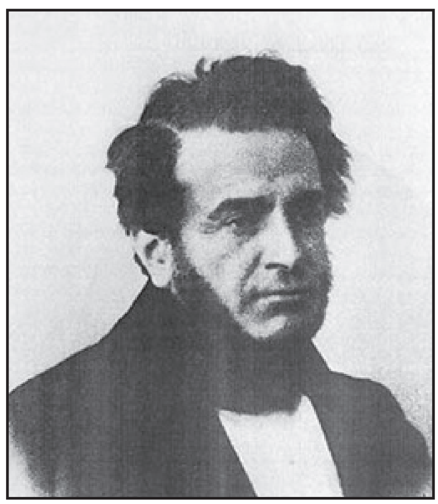
Not only mathematics was on the move in this period. The French Revolution had of course released an interest in rational philosophies, of which the *Encyclopédie* of Diderot and D'Alembert was a powerful example, and utopic socialist ideas soon took hold of people's imaginations. A leading charismatic figure here was Claude-Henri de Rouvroy, comte de Saint-Simon, who had fought in America side by side with Washington and on his return to France not only survived the Terror but made himself rich on land speculation, a fortune that he soon lost in his quixotic social enterprises. The other side of the medal must always be kept in mind: the Bourbon restoration in 1815 changed the political and social atmosphere in France quite radically.

Whereas the rationalism of the early revolutionary days never died in France, there was another somewhat opposite philosophical current that had a strong hold in other European countries, especially in Germany, *Natur Philosophie*. This had been started by the Dalmatian Jesuit Rutger Josip Boscovich (1711–1787) and although it had somewhat mystical undertones it had lasting (and not always negative) effects in the development of physics and mathematics in this period. Hans Christian Oersted, the discoverer of electromagnetic interactions, was a strong supporter, and even hard-headed scientists like Davy and Faraday followed Boscovich's ideas, albeit secretly. In Britain the poet Samuel Coleridge was very influential in propagating Pythagoric ideas on the mystical value of the real numbers, ideas that were enthusiastically supported by his friend, perhaps the most important mathematical physicist of the century, Sir William Rowan Hamilton (1805–1865).

In all these activities (except of course *Natur Philosophie*) Olinde Rodrigues took a part but because in almost

*This paper is based on material contained in the forthcoming book *Mathematics and Social Utopias in France: Olinde Rodrigues and his Times*, Simon Altmann and Eduardo L. Ortiz (eds), American Mathematical Society, History of Mathematics Series, Providence, RI, USA, 2005.

every case he was prevented by circumstances from taking a leading role, he became an elusive figure in the intellectual history of his period. He was a Jew and as such he benefited from Napoleon's concession of citizenship rights to the Jews but, although he was probably one of the most brilliant young mathematicians of his generation, the Bourbon restoration closed for him the possibility of an academic career. He became involved in finance, following on family tradition, but he soon was under the influence of Saint-Simon and enthusiastically embraced a career as social reformer, supporting the rights of women and of workers. Again here, however, he was displaced in the leadership of the Saint-Simonians by a far more charismatic character, Prosper Enfantin, to whom he had at one time coached in mathematics. He nevertheless kept doing some mathematics from time to time, and his papers would have been enormously influential had people paid attention to them. There is no doubt, for instance, that if his work had been properly understood the disastrous hundred years war between the quaternionists (followers of Hamilton) and the vectorialists (Gibbs *et al.*) would never had taken place. But in many biographical accounts of him he is merely presented as a social reformer, and his mathematical works were not properly recognized until decades after his death. Even in 1938 his compatriot, the brilliant mathematician Élie Cartan, did not have a clear idea of who he was: he referred to him as Olinde *and* Rodrigues, these two ghostly authors thus gatecrashing the mathematical literature for some considerable time.



Even the most elementary facts about Rodrigues had been covered in a cloud of misinformation, about his family, his education, even the date of his death. To correct this situation my collaborators and myself have investigated in the last few years primary sources that give a better and clearer picture about our man. We may now start at the beginning.

Family and education

The spelling of Olinde's surname led many to believe that his family origins were Portuguese rather than Spanish. To make matters worse, the substantial Sephardic Jewish community in Bordeaux, where Rodrigues was born on 6 October 1795, was universally referred to in France as the *Nation Portugaise*. We know for a fact, however, that Olinde's great-grandfather, Isaac Rodrigues-Henriques, was born in Spain between 1689 and 1691, and that he moved to Bordeaux, probably in the first quarter of the eighteenth century. Obviously, as all Jews that remained in Spain after their expulsion in 1492, Olinde's great-grandfather must have been a *marrano*, that is a converted Christian. Bordeaux was the most important French port and after its transfer from British to French suzerainty in 1453, many Spanish Jews emigrated to it and then reverted to Judaism. Olinde's father, Jean Isaac Rodrigues-Henriques, was an accountant who worked for fifteen years as a book-keeper (a responsible job, book-keepers often acting as cashiers) and then took the family to Paris in the late 1790's, where he worked as an exchange agent for a banker and published a manual on accountancy and banking. He also took a distinguished part in the *Grand Sanhedrin*, a council summoned by Napoleon in order to ascertain whether Jews where fit to become French citizens, which was finally agreed to be the case.

Before we get into the question of Rodrigues's education I must say a few words about his name. He had been registered at birth just as Benjamin Rodrigues, the 'Henriques' of the family name being dropped. In 1807, after Jews were allowed to become French citizens, regulations were established requiring them to modify their family names in order to avoid confusion through excessive repetition. Soon later, it was also required that Jews added a non-Jewish name to their given names, and in 1808 the Decree of Bayonne made them register with fixed given and family names. It is then that Rodrigues appears as Benjamin Olinde, the first of which names he soon dropped. His father was obviously a well-read man, since 'Olinde' is a character in Torquato Tasso's *Gerusalemme Liberata*, which had recently been translated into French. Despite the central role that Olinde's father had in the Jewish community, the family were not observant. (Olinde's wife, Euphrasie, was a Catholic.) On the other hand, they kept close ties with the members of the Parisian Jewish community originating from Bordeaux, many of whom were distinguished in the arts and finance. One of Olinde's sisters married a first cousin on the maternal side, Jacob Émile Pereire, who later became an important and wealthy financier and kept a very close relationship with Olinde throughout his life.

Thanks to Napoleon, Jews were now entitled to attend the state Lycées recently opened in Paris, and Olinde enrolled in 1808 at the *Lycée Impérial*. (Later his only, younger, brother also entered a Lycée, but none of his six sisters received similar instruction, although they were well educated by the standards of the time.) In 1811, Rodrigues got the first prize at the mathematical end-of-the-year competition, the second accessit going to his friend Michel Chasles, later a famous geometer. This was the second time that Rodrigues had won the competition, which was most important as we shall see. Rodrigues's mathematical teacher was C. Dinet, who was also a teacher at the *École Polytechnique* (where he had taught mathematics to the great Augustin-Louis Cauchy) and he was an examiner for the entrance to that school, for which purpose the competition mentioned was held. It is thus clear that Rodrigues was abundantly qualified to enter that prestigious establishment which, however, he never did. It cannot be thought that this was the case because of his Judaism, since we know at least two Jews that were admitted, Abraham Gabriel Mossé in 1798 and Myrtil Maas, a contemporary of Olinde, who was accepted in 1813. The *Polytechnique* was a military establishment, geared for the instruction of high powered military engineers but there was also the *École Normale Supérieure* oriented towards the training of teachers. Despite statements to the contrary in the literature (included mine) Olinde never even applied to become a *normalien*.

How, then, did Olinde become a fully blown mathematician? There is here a gap in our knowledge, but the circumstantial evidence is sufficient to sustain a very reasonable conjecture. We know that Rodrigues was so precocious that in 1812, when he was only 17, he was already a class assistant (*maître d'études*) in the mathematics course run at the *Lycée Napoléon* by his teacher, Dinet. In 1808 the University of Paris had been founded and it is entirely plausible that Dinet felt that Olinde was ready to dedicate himself to research rather than frequent the courses of the *Polytechnique*. In fact, not only was Rodrigues granted a *licence ès sciences* by the new Faculty of Sciences of Paris already in 1813, but also this work and five others were published in the *Correspondance sur l'École Impériale Polytechnique*, a publication devoted to the work of the School members. There is no other mathematician that has published as many papers in this journal, and it must be assumed that it was Dinet's patronage that permitted this to happen. These papers were published from 1815 to 1816, and we know that he gained the degree of *docteur ès sciences* from the University of Paris in 1815 (when he was not yet 20), so that it is clear that Rodrigues must had done mathematical research there for two or three years immediately after he left the *Lycée*. 1815, alas, was the year of the Bourbon restoration and with it the chances of an academic career for Olinde totally evaporated: there is evidence in fact that the educational bureaucrats alleged that parents would not permit their sons to be taught by Jews.

In the circumstances Rodrigues had to follow his father's line and dedicate himself to working in finance. For a period he collaborated with his friend Myrtil Maas who, finding himself in the same predicament, took employment in an insurance company, in which he created an actuarial committee. Both friends then produced the actuarial tables that would be used by all French insurance companies throughout the century, which were eventually published in 1860 and had seven editions until 1933. The core of Rodrigues's gainful work, however, was in financial matters, but he kept throughout his life his interest in mathematics. Not only we know that he read the mathematical literature but, from time to time, he produced some work of the highest quality, as I shall now discuss.

Olinde Rodrigues, the mathematician

Because of his non-mathematical activities Rodrigues's output is far from continuous, and can be roughly grouped in three clusters. The first one from 1813 to 1816 is largely related to his research for his licence and his doctorate. For his licence (1813) he had studied lines of curvature, obtaining a formula to relate the radius of curvature to the coordinates of the centre of curvature, which the great French differential geometer Gaston Darboux was to consider sixty years later as fundamental in the subject. In 1816 he published a paper in which he studied the motion of free objects by using the principle of least action in the calculus of variations, at that time a highly original piece of work which, precisely because even good mathematicians did not understand the importance of the least action method was ignored for decades. This work was not recognized until the Cambridge mathematician Edward Routh rediscovered it in the 1870's. The fate of this work is typical of that of Rodrigues: because he was not in academic circles and thus never had students that could drum up the merits of the master, he became the invisible man of nineteenth century mathematics.

Following on the work of his licence dissertation, he published some work on surfaces. Anticipating Gauss, he used a spherical mapping of a surface, studied the ratio of the areas of the corresponding surfaces, and arrived at a quantity later called the total Gaussian curvature, which he showed equals the product of the principal curvatures. This measure of curvature was credited to Gauss, although it appears not to have been known by the latter: an example of how Rodrigues was often robbed of recognition. He was luckier with another paper of 1816 on the attraction of spherical bodies, his doctoral research, that led to the only piece of work for which he ever got full credit. He had to use the so-called spherical harmonics, relatively new then (they had been invented by Adrien Legendre in 1784). The core, so to speak, of these functions are the *Legendre polynomials*. They were fairly well known in Rodrigues's time, but there were no closed formulae to derive them, and he invented an ingenious way to obtain one such, called the *Rodrigues formula* for the Legendre polynomials, used and named in this fashion to this day, although until Hermite discovered this work it was called the formula of Ivory and Jacobi, despite the fact that the priority for it is undoubtedly Rodrigues's. The procedure that Olinde devised to solve the problem, based on what he called *generating functions*, (which had already been used in statistical problems) was later found most useful in deriving similar formulae for many polynomials of interest in mathematical physics.

We must now jump a quarter of a century to reach Rodrigues's next cluster of five mathematical contributions (1838–1840), a quarter of a century that was nevertheless immensely productive in other ways: amongst other things, he became a man of business, with various degrees of success. Given this large gap, it is virtually impossible to guess Rodrigues's motivation for involving himself in this research, although it is quite clear that around 1838 and 1839 he became interested in combinatorial problems, on which he published four papers. By far, the most important of these is the last one. Clearly, Rodrigues was keeping abreast of mathematical research by reading both the *Liouville* and the *Crelle* journals, in which two papers appeared in 1838 on a combinatorial problem involving the inversion of permutations. Rodrigues solved the problems treated in these papers by defining again some generating functions, which had been so important in his work on the Legendre polynomials, and it is possible that he was inspired to do this work when he recognized the possibility of their use also in this case. Again, this paper was so assiduously ignored for more than a hundred years that repeated attempts appeared in the literature to solve the problem on which Rodrigues had said the final word. It was not until well into the second half of the twentieth century that this paper was rediscovered by L. Carlitz in 1970.

The next paper that we must now consider is one of the most remarkable pieces of work done by Rodrigues, in which he goes back to the subject of his very first paper in the *Correspondance*, rotations, but now of a sphere with fixed centre. The year is 1840 and all we know about him from the historians of the period, who ignored Rodrigues as a mathematician, is that in that year he was 'speculating at the Bourse,' although he was also much concerned about the legislation for the *Banque de France*. The story starts with one of the most prolific mathematicians that ever lived, the Swiss Leonhard Euler (1707–1783). In 1775, when he was already blind, he published in St. Petersburg, where he lived most of his life, two papers about rotations. The rotation of a sphere around its centre, for instance, is fully denoted by a line, the axis of rotation, and by an angle about that line, the angle of rotation. Euler proved that two such rotations when performed one after the other produce a third rotation, called their 'product', and as a result he described rotations in terms of three angles, called the *Euler angles*. Rodrigues went much further, because, given the axis and angle of each rotation he produced a geometrical construction that determines the angle and axis of the product rotation. I illustrate this in Fig. 1, where the first rotation effected is one with axis **a** and angle α, the second with axis **b** and angle β. The product rotation is around **c** by γ. Of course, it required considerable ingenuity for Rodrigues to have discovered that construction, but I shall leave aside his proof since I show this picture to call attention to the fact that the rotations appear there in terms of their half angles. This apparently minor result is however of momentous importance, and it was the cause of one of the greatest controversies in the history of mathematics.

By using spherical trigonometry in a straightforward manner Rodrigues was able to replace each rotation by a set of four algebraic parameters, in such a way that, given the two sets for the two successive rotations, he was able to obtain the same set of four parameters for the product rotation, a truly remarkable result. These are now the essential tool in the modern work on rotations, and the four parameters that Rodrigues introduced are universally used in studying spin, in describing molecular structures, in spacecraft kinematics, in robotics, even in ophthalmology, having correctly replaced the use of the awkward Euler angles. The importance of Rodrigues's contribution,

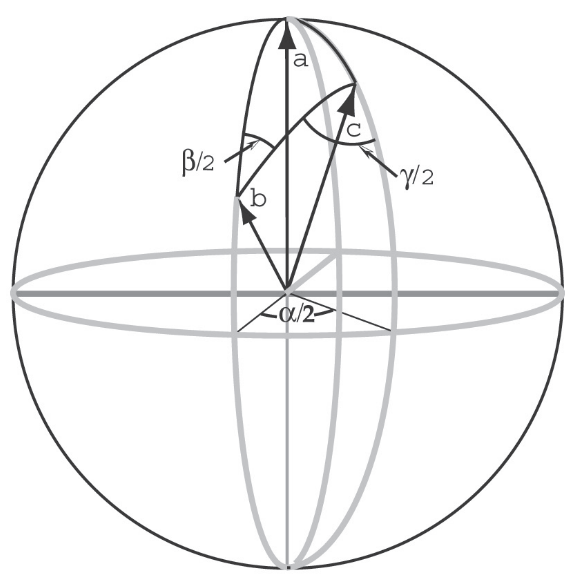


Fig. 1

however, took some time to be properly recognized. The very successful treatise on the dynamics of rigid bodies by Routh, which reached eight editions, the first one published in 1860, contained no reference to Rodrigues until the third edition in 1877, when he proposed that the theorem on the composition of rotations be named after Olinde Rodrigues. Indeed, all the successive editions, up to the last one in 1930, contain a section labelled 'Rodrigues's Theorem'. Such recognition, however, was not readily granted. When the famous German mathematician Felix Klein published his *Vorlesungen über das Ikosaeder* in 1884 he gave Rodrigues some, soon forgotten, partial credit: he grants Rodrigues the discovery of his four parameters and acknowledges that they were unknown to Euler, but he asserts that the latter had used three analogous parameters by replacing the four of Rodrigues by the quotient of his last three by the first. This is a total misrepresentation of the facts, since, as I have said, Euler never used half angles. Every opportunity was taken to bring in Euler and push the inconvenient Rodrigues away: Schoenflies and Grübler, Klein's collaborators, writing in the most prestigious mathematical encyclopaedia of its time, say in relation to these formulae: 'These formulae are quite often treated in connection to Euler. We owe a first essential advance to O. Rodrigues.' The bible of rotation theory, the monumental work by Klein and Sommerfeld, gives those four parameters without any attribution. Euler's great name pushes Rodrigues off: Fig. 1, which is the corner stone of mathematical crystallography, is universally called the *Euler construction*, although Euler was never even near producing it.

The work of Rodrigues in 1840 is historically even more important, because it should have been used by any unprejudiced mathematician to unravel one of the most extraordinary blunders in the history of mathematics. However, because this blunder was embedded in one of the most important and ingenious creations of the century, people firmly shut their eyes to it. Sir William Rowan Hamilton, the man who in 1865 was to be ranked as the greatest living scientist by the American National Academy of Sciences, had also constructed, three years after Rodrigues, some objects composed of four elements that he named *quaternions*. He obtained them, not from a geometrical point of view, but in an a priori way through algebra. (Remember that he was under the somewhat mystical approach of *Natur Philosophie*.) This work, from the algebraic point of view, was outstanding, but from the point of view of rotations it was an unmitigated disaster that caused a century of confusion and strife. The reason for this is that, having discovered the quaternions algebraically, Hamilton had to interpret them geometrically. He realized of course that his quaternions were related to rotations, but he used two principles that had unfortunate consequences. The first was to give priority to the algebra over the geometry, whereas it was geometry, which had a strong tradition in his mathematical circle, that had allowed Rodrigues to keep his feet firmly on the ground. The second was to use only full and not half angles, for which he had very sensible reasons, which nevertheless were to be contradicted by the facts a century later. He never read Rodrigues's paper and he never derived a product rule for the rotations as Olinde had done, but only in a different very indirect way. Cayley, who published this latter work before Hamilton did, not only acknowledged Rodrigues (the only one to do so at the time) but confessed that he did not understand why their convoluted algebraic method gave the same results that Olinde had obtained directly, a question that was not cleared up for more than a hundred years.

In order to understand the nature of Hamilton's blunder a great deal of detailed work is required, but I shall attempt here to present a rough argument to give an idea of the problem. On using the modern vector notation, a quaternion is defined as a pair of a scalar a and a vector, say: $[a, A\mathrm{i} + B\mathrm{j} + C\mathrm{k}]$. A *unit quaternion* may be defined as $[a, A\mathrm{i}]$, with $a^2 + A^2$ unity. Hamilton here fell into two traps. One is that he did not realize that the vector in the quaternion is an axial and not a polar vector, an understandable failure, since he was inventing vectors at the same time as doing his quaternion work, and the distinction in question was not known until late in the century. It was this failure, however, that created untold problems when pursuing Hamilton's programme of defining vectors via quaternions. The highly respected mathematician Marcel Riesz, who first pointed out this error in 1958, qualified Hamilton's interpretation as 'grossly incorrect'. The second trap is that Hamilton noticed that a quaternion $[\cos\alpha, \sin\alpha\,\mathbf{i}]$, acting on a vector normal to $\mathbf{i}$ 'rotates' it by the angle α. and interpreted this as a rotation by α around the axis i of any vector normal to $\mathbf{i}$. This, however, is not at all a rotation of the vector, but an artefact resulting from the product of two rotations, as I demonstrated in 1986. It was clear in any case that there was something wrong in Hamilton's interpretation, since the same so-called 'rotation', acting now on a general vector, does not transform it into its rotated vector, a result that of course Hamilton knew, but preferred to ignore. In fact, he often took the primary definition of a quaternion as an operator that acting on a vector rotates it into another vector or, what is the same, as the quotient of two vectors, an erroneous statement that still appears in the mathematical literature, although it is now accepted to be wrong. It must be understood, however, that Hamilton had excellent reasons to fall into a trap. His 'rotation by α', $[\cos\alpha, \sin\alpha\,\mathbf{i}]$, does not change sign when 2π is added to the alleged angle of rotation, whereas if we had, after Rodrigues, $\alpha/2$ in it, it would. Such behaviour would have been unacceptable to Hamilton and, in fact, it took the best part of a century to understand that this curious change of sign was required by the topology of rotations, a result that became crucial in quantum mechanics.

Rodrigues's paper goes much further than I have described. When symmetry operations are combined, their set forms what mathematically is called a *group*. Rodrigues considered not only the rotation but also the translation group, the aggregate of which forms the so-called *Euclidean group*. Translations appear to be totally distinct from rotations, but Rodrigues realized them by producing infinitesimal rotations around infinitely distant rotation axes. It was to take more than half a century before this remarkably clever idea was again used.

As we have seen, even at this time Rodrigues had not totally abandoned mathematics, and he still read the mathematical literature. He had done nothing on the lines of this paper, however, since 1814, and it is a wonder why he returned to the subject. Geometry, however, had been a strong influence in his mathematical upbringing: Monge had created a powerful school, of which Michel Chasles was an important member. A school friend of Rodrigues, Chasles had continued their acquaintance: we know that they met socially in the 1830's and it is possible that his influence revived Rodrigues's thoughts on rotations. It is important to remember that precisely at that time, in 1830, Chasles produced one of the fundamental results in mechanics, called to this day the Chasles theorem, namely that the most general motion of a rigid body is a translation combined with a rotation around a fixed point. It is highly probable, therefore, that this problem was discussed by the two friends, and that it remained in Rodrigues's mind.

Three years after the seminal 1840 paper Rodrigues produced his final cluster of three mathematical papers (1843–1845). The first two, are concerned with trigonometrical problems, and are followed in 1845 by a short note on continued fractions, his last mathematical paper.

Before we leave this brief discussion of Rodrigues's mathematical works, it is important to mention his writing style. No one who has read the original works of the mathematicians of his period, however great, can help experiencing a sense of relief in reading Rodrigues. His language is concise and entirely free from jargon, and his form of exposition is amazingly clear. Amongst those who have had that experience, the word 'gem' often recurs. The contrast with Hamilton, in particular, is staggering, and one can see why. Hamilton wrote some times starting from pre-conceptions, and when these proved a trap he refused to abandon them. Pre-conceptions were never part of Rodrigues's mathematical writing, which could thus flow from fact to fact without impediment. The opposition of French rationalism with the *Natur Philosophie* quasi-mysticism of Hamilton and his followers could not have a better example than the comparison of these two authors.

Olinde Rodrigues banker, social reformer, Saint-Simonian

Two interests were central to Olinde for most of his life: financing, and how to use it for social purposes. Following on his father, he became a free-lance broker at the Bourse and in 1823, was the director of the *Caisse Hypothécaire* (a bank concerned with mortgages) at the rue Neuve-St-Augustin. In that year his life changed: Saint-Simon, by now destitute, attempted to commit suicide but only succeeded in becoming an invalid. A common friend brought Rodrigues as a man of means that might help him. Helped he did and more, because he became mesmerized by the charismatic Saint-Simon and collaborated with him until his death in 1825 in writing up his last work, *New Christianity*. After Saint-Simon's death Olinde, with the help of his younger brother Eugène, created a quasi-religious sect with strong socialist undertones. He soon, however, was displaced as its leader by Prosper Enfantin, who had been his private mathematics pupil. Enfantin was a hugely charismatic man whose ideas were very different from Rodrigues's: he preached free love and held sway on the females of the congregation, including Euphrasie, Olinde's attractive wife. Rodrigues opposed the loose morality supported by Enfantin, that caused the sect to begin to disintegrate already in 1831. Although Olinde tried to be faithful to his new master, he himself seceded in 1832. In that year there was a notorious trial of Saint-Simonians, accused of the corruption of public morals and illegal association. Enfantin was found guilty on both charges and sent to prison for a year, but Rodrigues was only charged on the second offence and merely fined.

Despite the debacle of the sect, Rodrigues remained a social reformer at heart. Given his knowledge of finance he devoted a great deal of work to promoting the use of finance for social purposes, such as the construction of railways, since he understood the importance of transport to improve the standard of living of the working classes. (His cousin, Émile Pereire and many other Saint-Simonians became very active in the development of the French railways.) In 1840 Olinde founded a journal, *Le Patriote*, to promote the rights of workers, proposing their right to holidays, then such a novel idea that was not adopted until 1912. With his friend Gustav d'Eichthal he fought for the abolition of slavery, that came about in 1848. A very important passion of Rodrigues was that of improving the condition of women, fighting for their legal rights, including the right to vote. Rodrigues received in fact recognition by women often writing to the press to acknowledge this work.

His idealism was never extinct: in February 1848, when at 53 by the standards of the time he was a fairly old man, he mounted the barricades in Paris and in April of that year he was found in London, haranguing the multitudes in support of the Chartists who were demonstrating in sympathy with the French revolutionaries. He died on Wednesday 17 of December 1851, although many biographical notes about him give erroneously the date as 26 December 1851. Many of the short biographical notes about him merely describe Rodrigues as a social reformer, totally ignoring his mathematical work. Even with the Saint-Simonians he did not fare better. Enfantin and his followers were very conscious about their historical position and they prepared several archives at present in various Parisian libraries with carefully copied documents that contain no reference whatever to Olinde Rodrigues. Hardly ever such a worthy man has been so badly treated by history.