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EDITOR: *Gazeta de Matemática, Lda.* ADMINISTRADOR: *A. Sá da Costa*

REDACTORES: *J. Gaspar Teixeira, J. Morgado e J. da Silva Paulo*

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On the condition $(xy)^2 = x^2y^2$ in nonassociative rings

by **José Morgado**

Instituto de Matemática, Universidade Federal de Pernambuco, Brasil

1. It is well known that a group G satisfying the condition

(1) $(xy)^2 = x^2 y^2$ for all x, y in G

is necessarily commutative.

It is also well known that if G is a group such that

(2) $(xy)^i = x^i y^i$ for three consecutive integers i and for all x, y in G,

then G is necessarily commutative ([1], p. 31, exercise 4).

However, the ring-theoretic analogues of these group-theoretic results do not hold.

Thus, McCoy ([2], p. 15, example 6 and p. 31, exercise 7) gives an example of a noncommutative ring satisfying the condition (1) and E. C. Johnsen, D. L. Outcalt and A. Yaqub [3] give an example of a noncommutative ring satisfying the condition (2).

In [3], the authors prove that *if* R *is any nonassociative* (i. e., not necessarily associative) *ring with identity satisfying the condition (1), then* R *is commutative.*

The purpose of this note is to generalize this result.

2. Let us recall that an element a of a multiplicative groupoid G is said to be an *associative element*, if one has

$$ax \cdot y = a \cdot xy, \quad xa \cdot y = x \cdot ay$$

and

$$xy \cdot a = x \cdot ya$$

for all x, y in G. The element a is said to be *cancellable*, if for all x, y in G each of the equations $ax = ay$ and $xa = ya$ implies $x = y$.

We are going to state the following

Theorem: *Let* R *be any nonassociative ring satisfying the condition*

$$(\mathrm{x}\,\mathrm{y})^2 = \mathrm{x}^2\,\mathrm{y}^2 \textit{ for all } \mathrm{x}, \mathrm{y} \textit{ in } \mathrm{R}.$$

If for every $(\mathrm{x}, \mathrm{y}) \in \mathrm{R} \times \mathrm{R}$ *there is in* R *some element* e *which is associative and cancellable in the multiplicative subgroupoid of* R *generated by the set* $\{\mathrm{x}, \mathrm{y}, \mathrm{e}\}$, *then* R *is commutative.*

Proof: Indeed, one has obviously

$$(x(y+z))^2 = (xy+xz)^2 = xy \cdot xy + \\ + xy \cdot xz + xz \cdot xy + xz \cdot xz$$

for all x, y, z in R.